Magnetic Fluctuation-Induced Particle Transport and Zonal Flow Generation in MST

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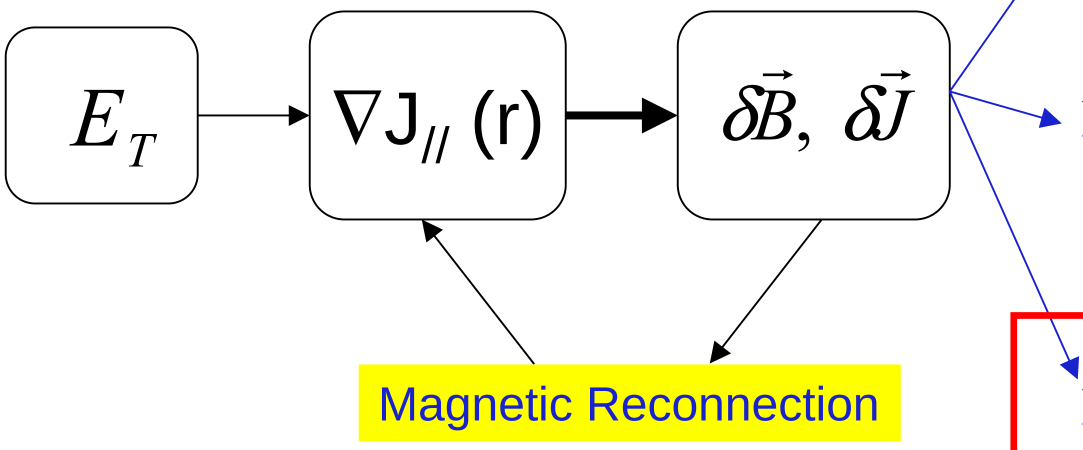
Introduction

Magnetic and Current Density fluctuations play an important role in plasma relaxation in the Reversed Field Pinch as well as tokamak configurations



Hall Dynamo $\dfrac{<\delta J \times \delta B>}{ne}$

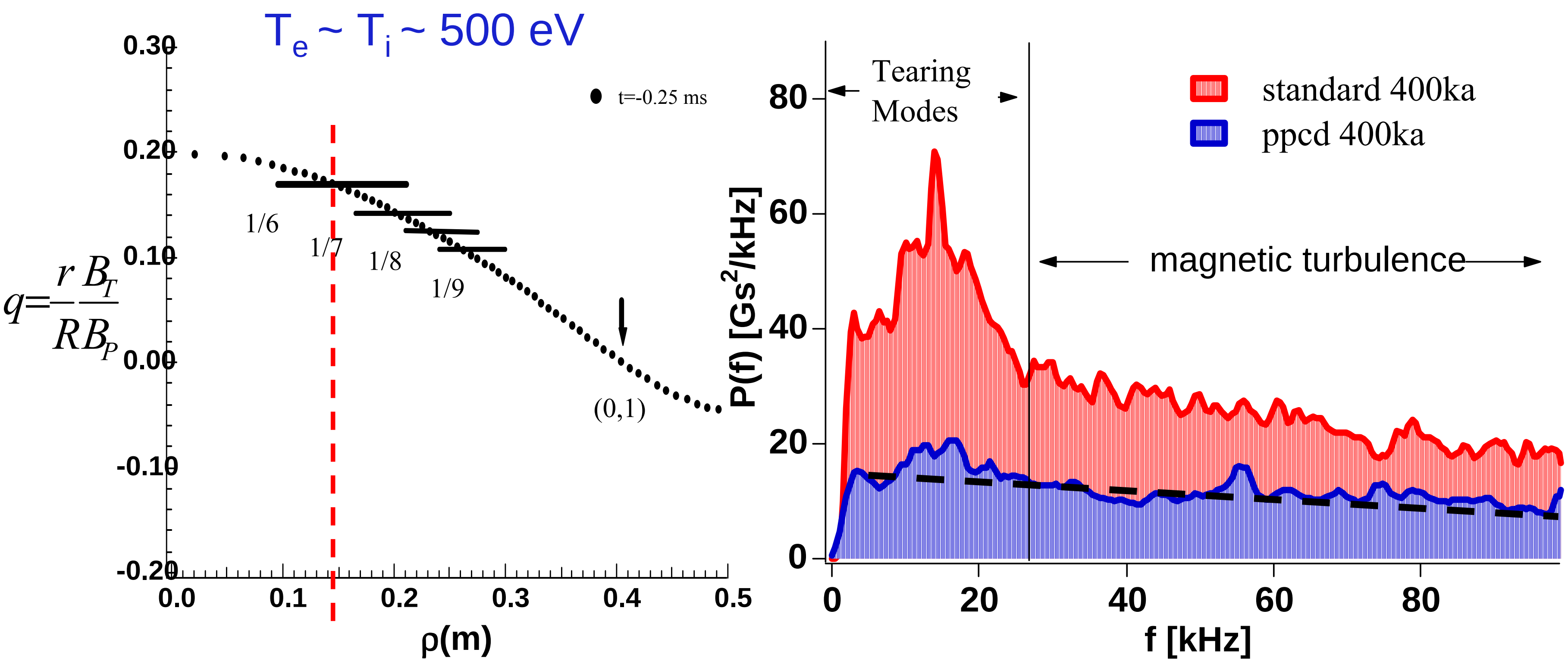
Momentum Transport $<\delta J \times \delta B>$

Particle Transport $\dfrac{<\delta J_{//} b_r>}{eB}$

- Maxwell Stress $<\dfrac{1}{r}\tilde{b}_r \dfrac{\partial}{\partial r} r\tilde{b}_\theta>$

All processes coupled through δJ and nonlinear mode interactions

q Profile and Core Magnetic Fluctuation Spectrum



Tearing modes and broadband magnetic turbulence

Magnetic Fluctuation-Driven Particle Flux

Fluctuation-Induced flux

$$\Gamma_{i,e} = \Gamma_{i,e}^{\ es} + \Gamma_{i,e}^{\ em} = \frac{< \tilde{n}_e \tilde{E}_\perp >}{B_0} \pm \frac{< \tilde{j}_{//i,e} \tilde{b}_r >}{eB_0}$$

↑ **Electrostatic** ↑ **Magnetic**

$$\Gamma_i^{em} = \frac{< \tilde{j}_{//i} \tilde{b}_r >}{eB_0}$$

$$\Gamma_e^{em} = \frac{- < \tilde{j}_{//e} \tilde{b}_r >}{eB_0}$$

$$non-ambipolar\ flux: \quad \Gamma_q = \Gamma_i - \Gamma_e = \frac{\langle \tilde{j}_\parallel \tilde{b}_r \rangle}{eB_0}$$

Radial Charge Transport $\quad j_r = e\Gamma_q$

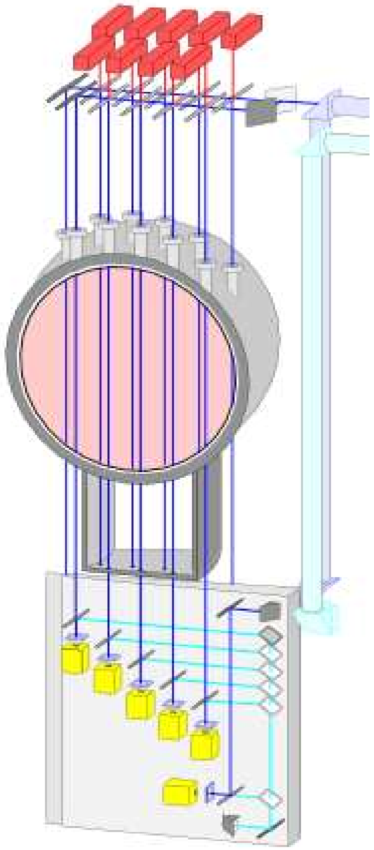
Fast polarimeter measures core mean and fluctuating *B & J*

Faraday rotation angle $\Psi \sim \int n\mathbf{B}\cdot\mathbf{dl}$

$$\delta\Psi = c_F \int n_0 \delta\vec{B}\bullet d\vec{l} + c_F \int \delta n \vec{B}_0 \bullet d\vec{l}$$

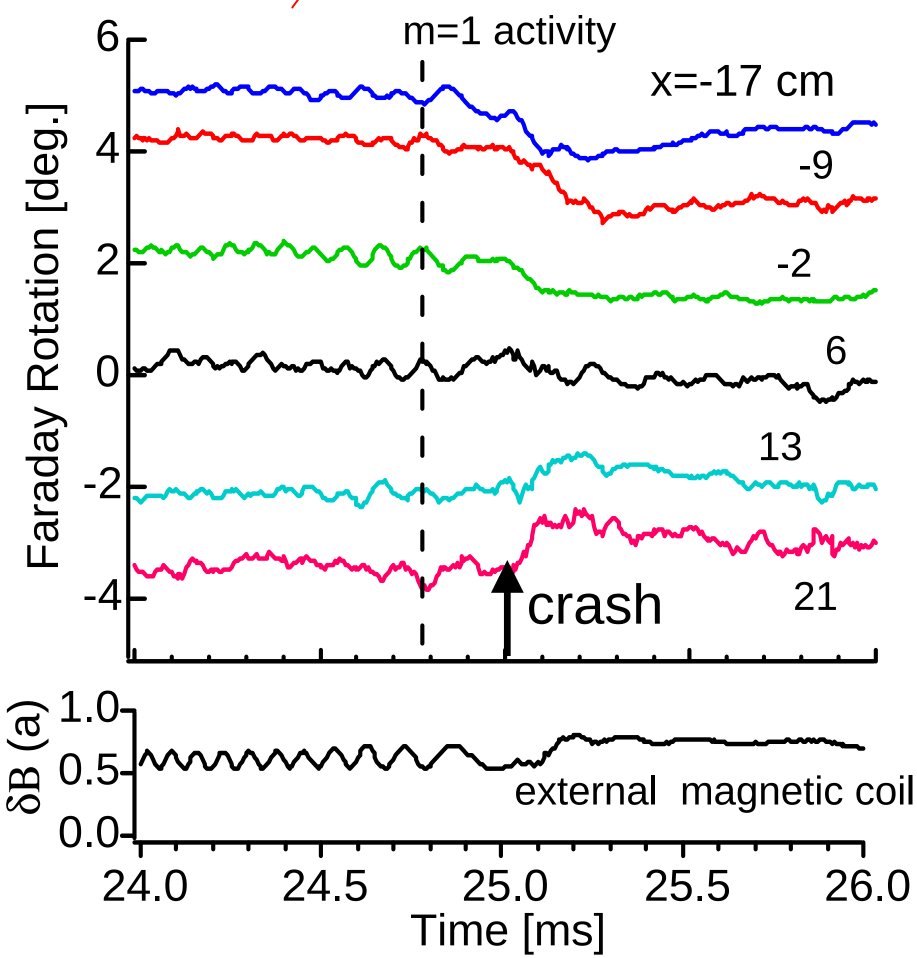
~ 0

11-chord FIR laser



+

32 magnetic coils toroidal array



$$\delta\Psi \approx c_F \int n_0 \delta\vec{B}\bullet d\vec{l}$$

$$\delta\overline{B} = 33 \text{ [Gauss]}$$

Current Fluctuation Measurement Method

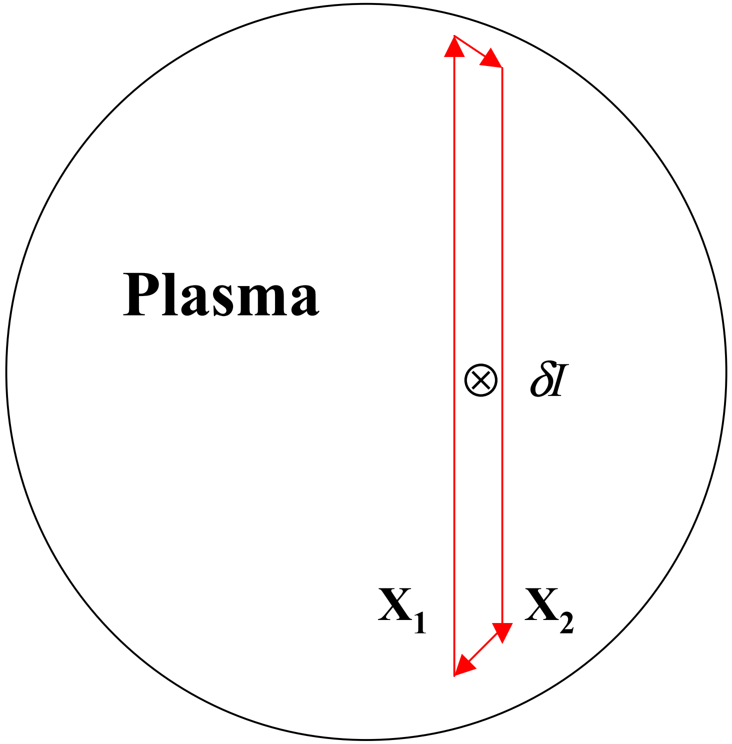
$$\text{Ampere's Law}: \oint_L \delta\vec{B} \bullet d\vec{l} = \mu_0 \delta I$$

Faraday Rotation Fluctuation:

$$\delta\Psi = c_F \int n_0 \delta\vec{B} \bullet d\vec{l} \approx c_F \overline{n}_0 \int \delta\vec{B} \bullet d\vec{l}$$

$$\oint_L \delta\vec{B} \bullet d\vec{l} \approx \left[\int \delta B_z dz\right]_{x1} - \left[\int \delta B_z dz\right]_{x2}$$

$$\approx \mu_0 \delta I_\phi = \frac{\delta\Psi_1 - \delta\Psi_2}{c_F \overline{n}_0}$$



Loop between polarimeter chords is equivalent to a Rogowski coil measurement

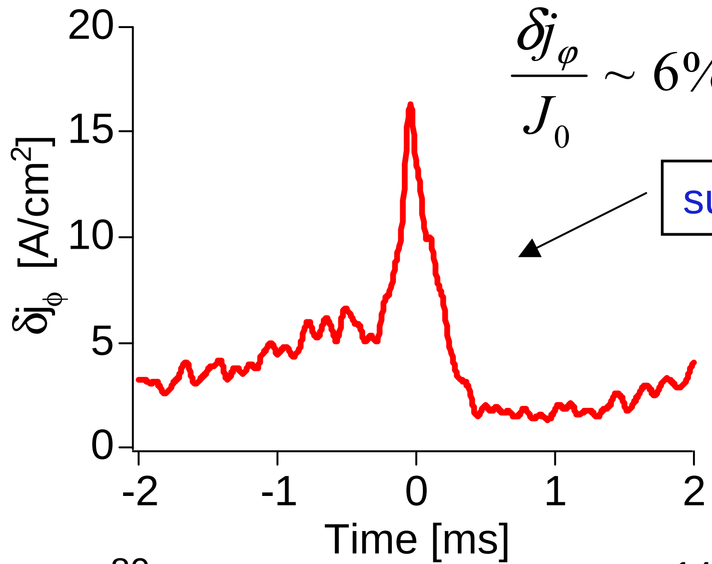
Ding, Brower et al. PRL (2003)

Measured Magnetic and Current Density Fluctuation Profiles

$$\frac{\delta j_{\varphi}}{J_0} \sim 6\%$$

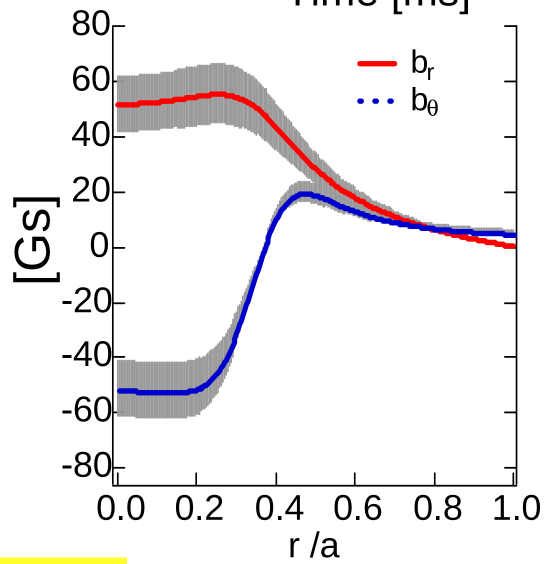
(m,n)=(1,6) tearing mode

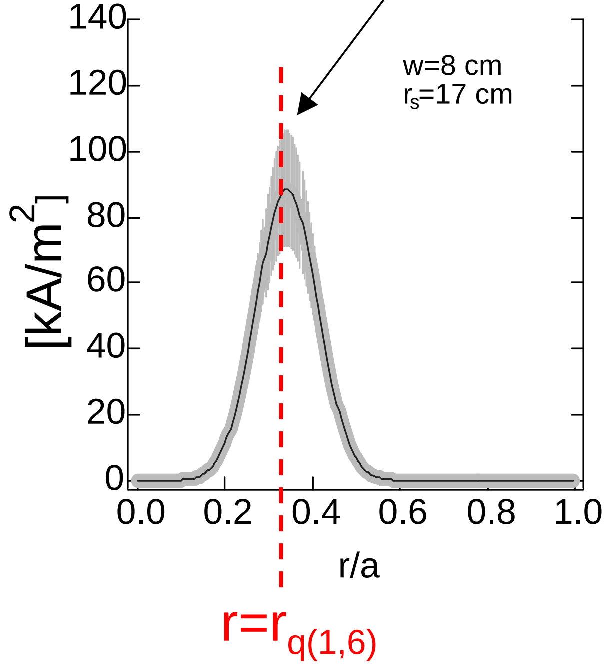
surges at sawtooth crash



spatially localized in core,

peaks at resonant surface





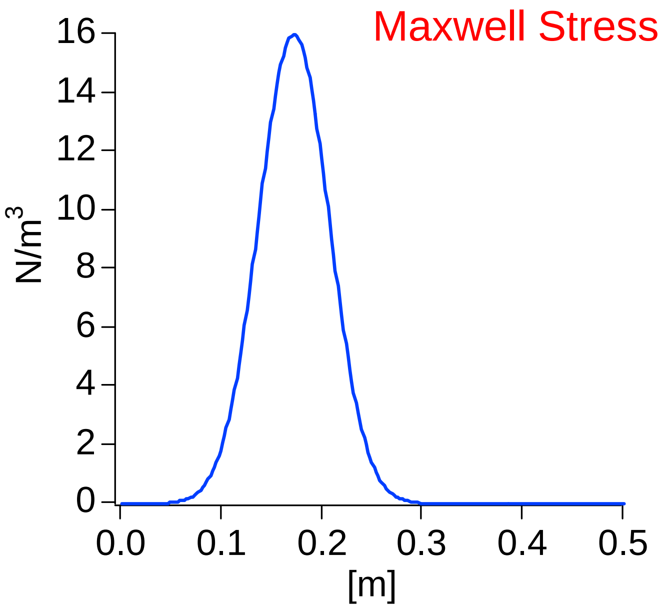
$$\frac{\delta B}{B} \sim 1\%$$

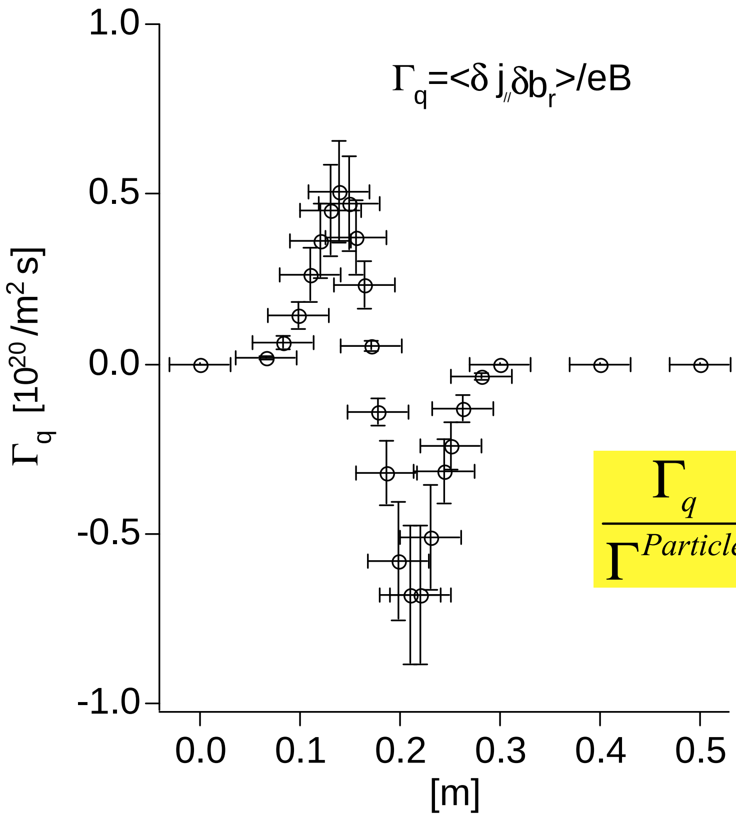
Measured Charge Flux at sawtooth crash in MST

$$\Gamma_q = \frac{<\tilde{j}_{//}\tilde{b}_r>}{eB} = \frac{1}{eB}\frac{R}{nB}(\vec{k}\bullet\vec{B})<\frac{1}{r}\tilde{b}_r\frac{\partial}{\partial r}r\tilde{b}_\theta> = \frac{1}{eB}\frac{B_T}{B}(1-\frac{m}{nq(r)})<\tilde{j}_\phi\tilde{b}_r>$$

where $\nabla\times\delta\vec{B} = \mu_0\delta\vec{J}$ and $\frac{|r-r_s|}{r_s} << 1$ *and* $\langle....\rangle$ *denotes flux surface average*

$$\frac{1}{\mu_0}<\frac{1}{r}\tilde{b}_r\frac{\partial}{\partial r}r\tilde{b}_\theta> = \frac{1}{\mu_0}<\tilde{j}_\phi\tilde{b}_r>$$





$$\frac{\Gamma_q}{\Gamma^{Particle}} \leq 1\%$$

Charge flux is radially localized and changes sign across resonant surface

Charge Transport and Radial Electric Field

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \vec{J} = 0, \quad \nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad \varepsilon_0 \frac{\partial E_r}{\partial t} = e(\Gamma_r^i - \Gamma_r^e)$$

$$\frac{< \tilde{j}_{//} \tilde{b}_r >}{B} \longrightarrow$$ 1~4 [A/m^2] at the core (FIR Faraday)

$$\Delta \tilde{E}_r = \int \frac{< \tilde{j}_{//} \tilde{b}_r >}{\varepsilon_0 B} dt$$

Leads to a huge electric field, ~50 MV/m in core

However, shielding occurs due to ion polarization current

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \vec{J} = 0, \quad \nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0 \varepsilon_\perp}$$

$$\varepsilon_0 \varepsilon_\perp \frac{\partial E_r}{\partial t} = \frac{< \tilde{j}_{//} \tilde{b}_r >}{B}$$

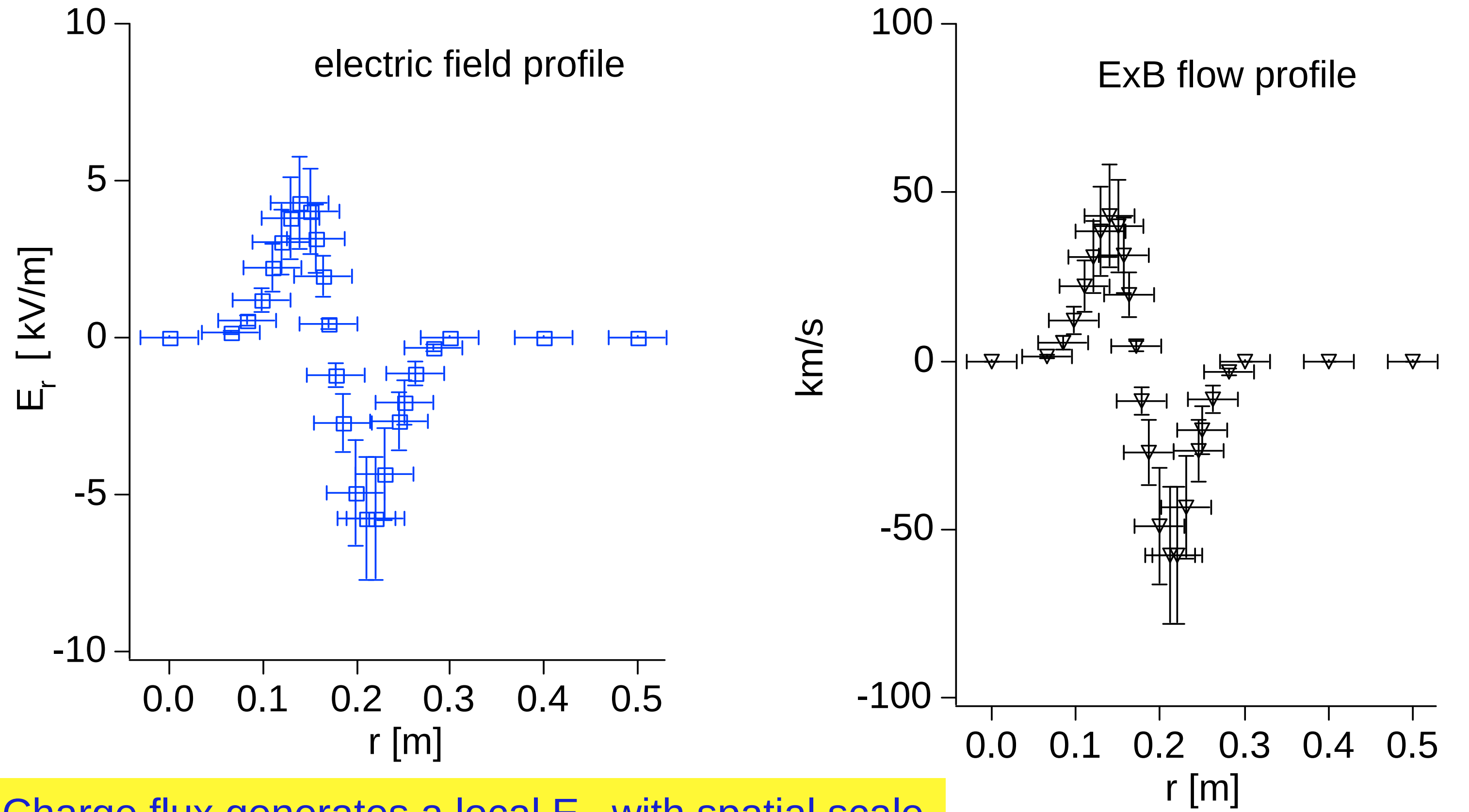
$$\varepsilon_\perp = 1 + \left(\frac{c}{V_A}\right)^2$$

$$\Delta E_r = \left(\frac{1}{1 + \frac{c^2}{V_A^2}} \right) \int \frac{< \tilde{j}_{//} \tilde{b}_r >}{\varepsilon_0 B} dt \approx \left(\frac{V_A}{c} \right)^2 \int \frac{< \tilde{j}_{//} \tilde{b}_r >}{\varepsilon_0 B} dt$$

At reconnection, a radial electric field is established due to non-ambipolar transport,

but electric field is reduced by 10^4 due to shielding by the ion polarization drift.

Localized Radial Electric Field and ExB Flow



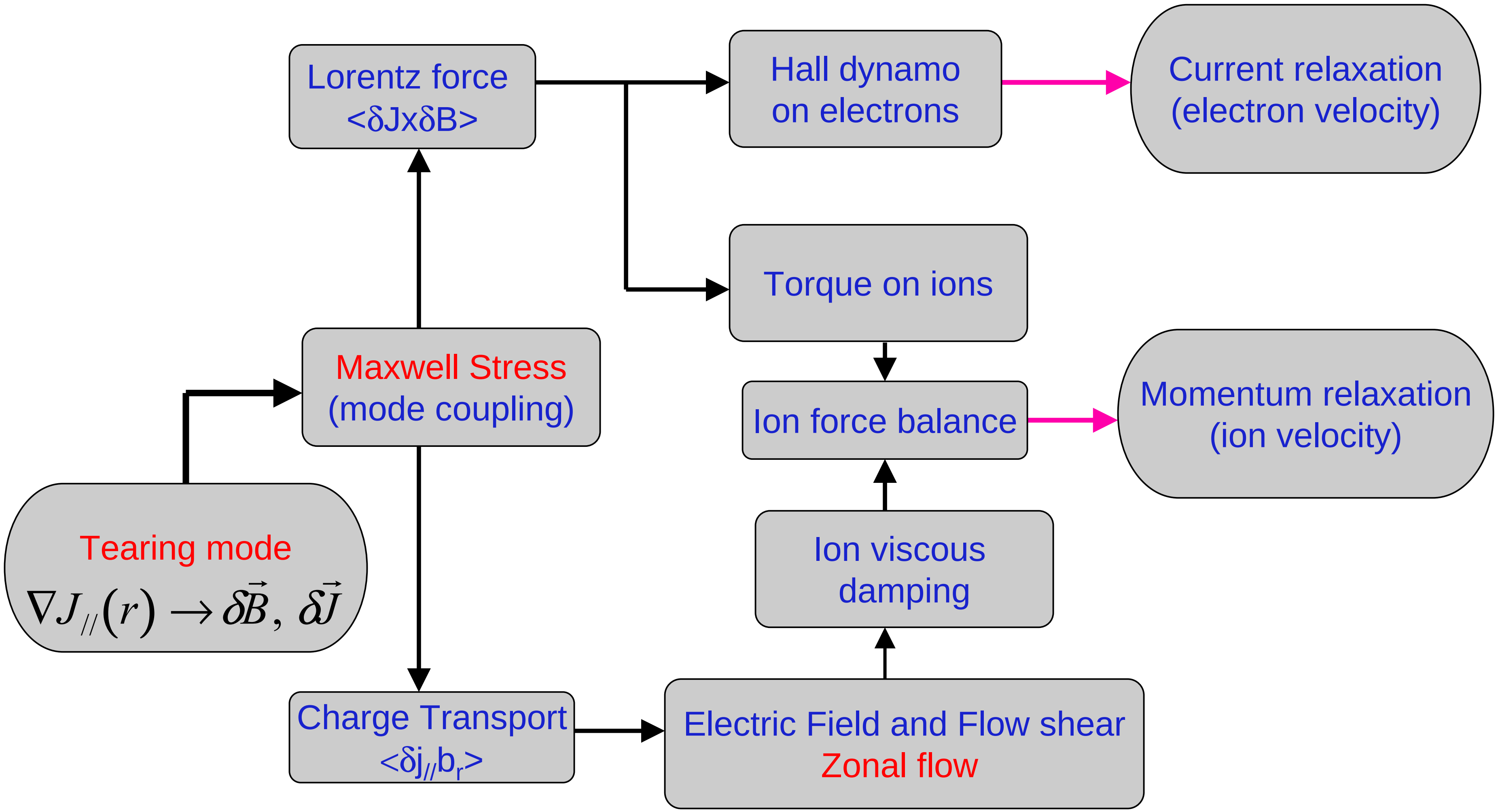
Charge flux generates a local E_r with spatial scale ~5 cm that changes sign across resonant surface

(1) ExB generates flow and flow shear (which may strongly damp the mean flow)

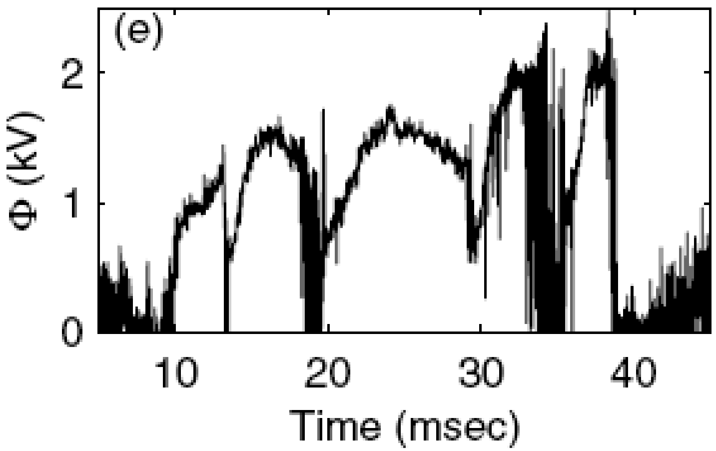
(2) Flow is toroidally and poloidally symmetric (m=0,n=0) *zonal flow* driven by resistive tearing modes

Summary

Measurements indicate the following coupled relaxations:



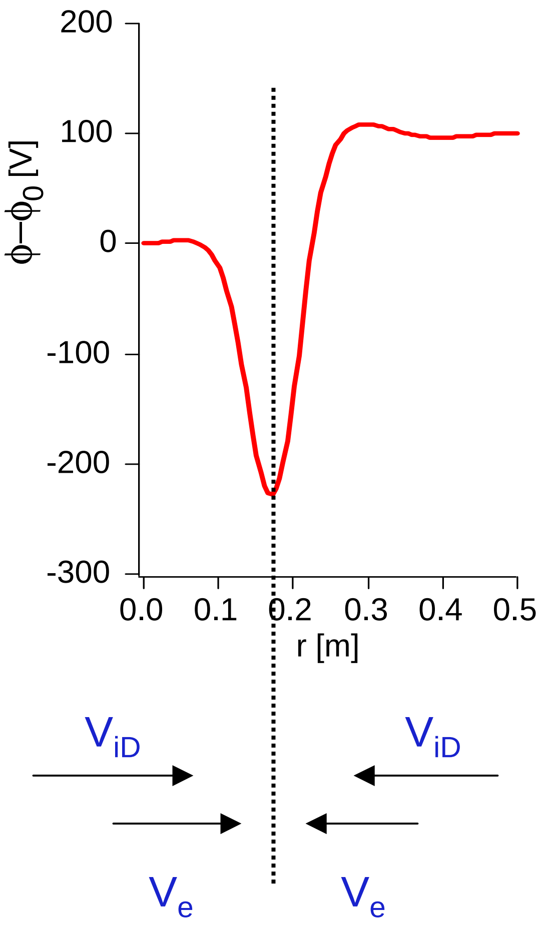
Evidence of potential structure ?



Potential measurement by Heavy Ion Beam Probe (HIBP)

Lei,J, et al, PRL, ***89,****275001(2002)*

Possible Effect of Electrical Field on Plasmas (Open Questions)



A potential well is formed near the reconnection layer

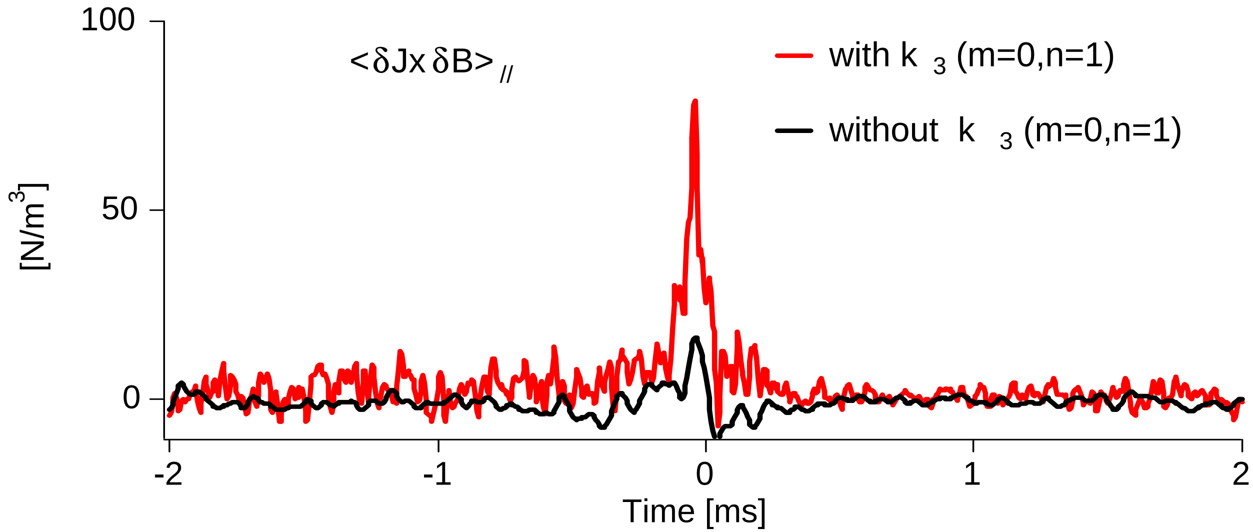
Questions not clear to me:

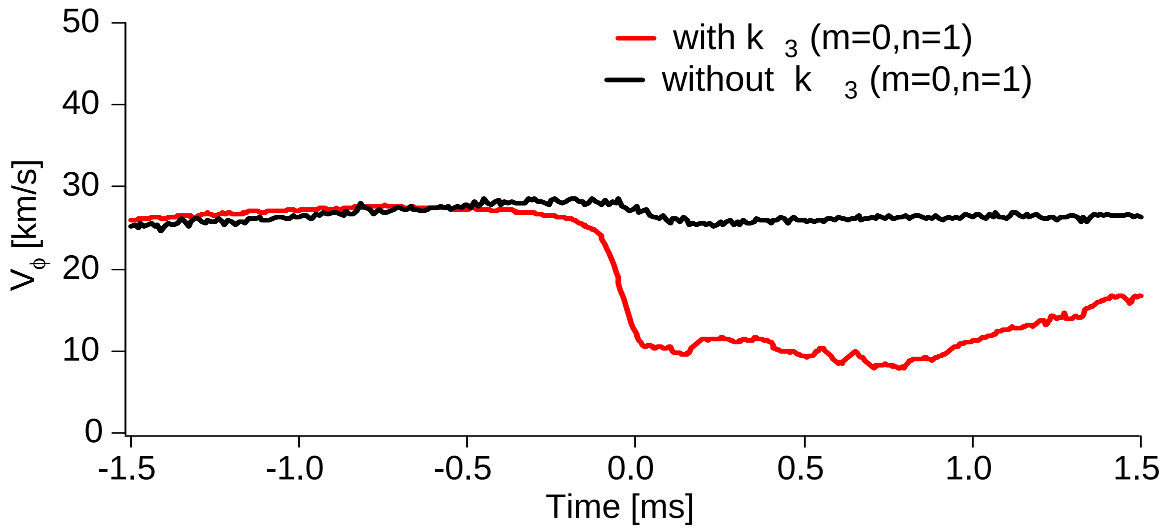
(1) Where does electric field energy come from?

(2) Do ions and electrons gain energy from the field?

(3) Where does flow energy go?

Momentum transport and nonlinear torque





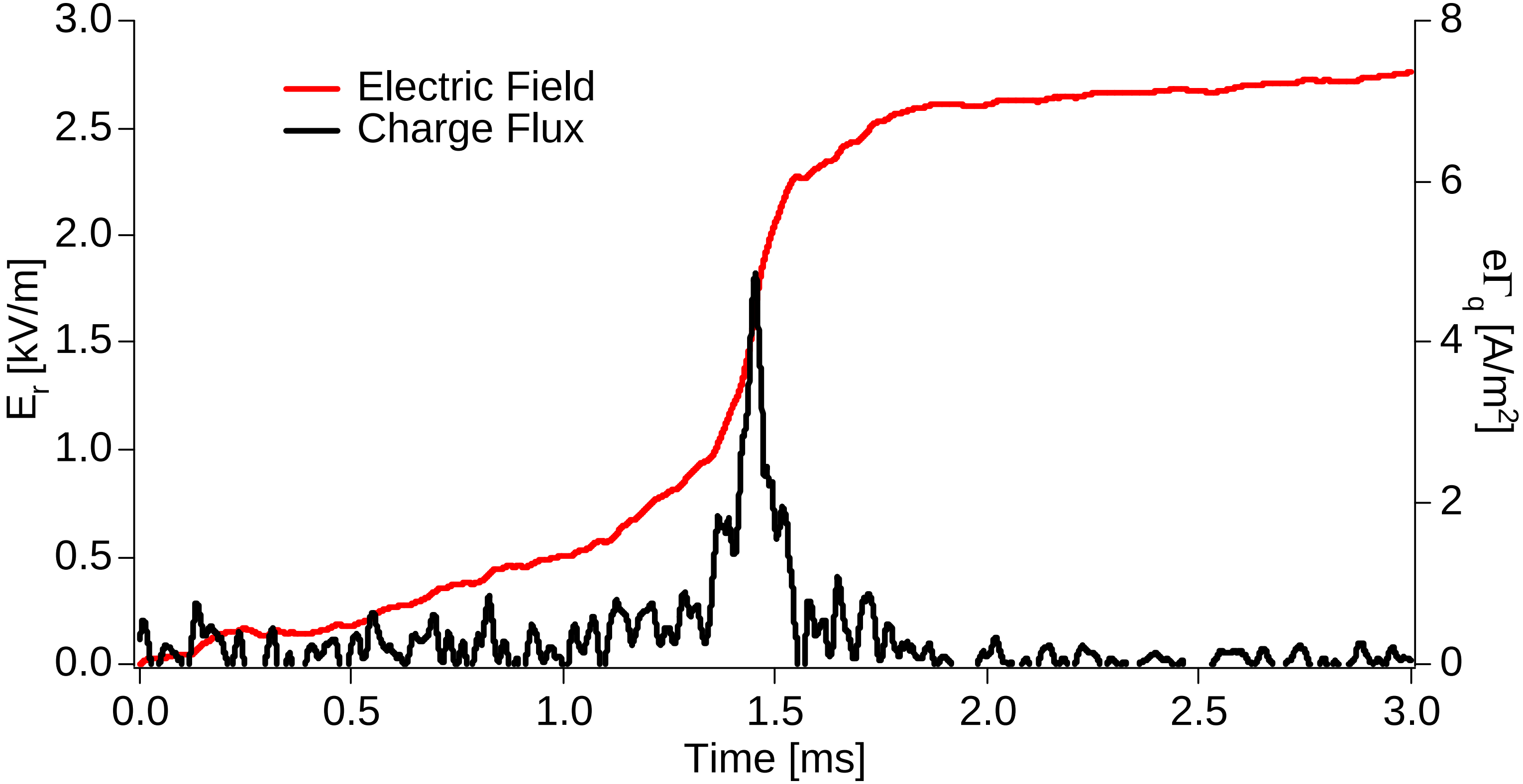
$$\vec{k}_1 \pm \vec{k}_2 = \vec{k}_3$$

$$\begin{pmatrix} 1 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

<δJxδB> and charge transport observed when three modes are coupled.

Dynamics of Radial Electric Field over Sawtooth

$$E_r \sim \left(\frac{V_A}{c}\right)^2 \int e\Gamma_q dt$$



What dissipates the electric field or dissipates ExB flow after the sawtooth crash??

Perpendicular Momentum Balance Equation

$$\frac{\rho}{B}\frac{\partial V_{E\times B}}{\partial t}-\frac{\mu_{\perp}^{*}}{B}\nabla^{2}V_{E\times B}=\frac{<\tilde{j}_{//}\tilde{b}_{r}>}{B}$$

ρ is mass density and μ^{*} is the classical viscosity coefficient

$$\frac{\rho}{B^{2}}\frac{\partial\overline{E}_{r}}{\partial t}-\frac{\mu_{\perp}^{*}}{B^{2}}\nabla^{2}\overline{E}_{r}=\frac{<\tilde{j}_{//}\tilde{b}_{r}>}{B}$$

ion viscosity $\mu_{\perp}^{*}=\frac{3nkT_{i}}{10\omega_{ci}^{2}\tau_{i}}$

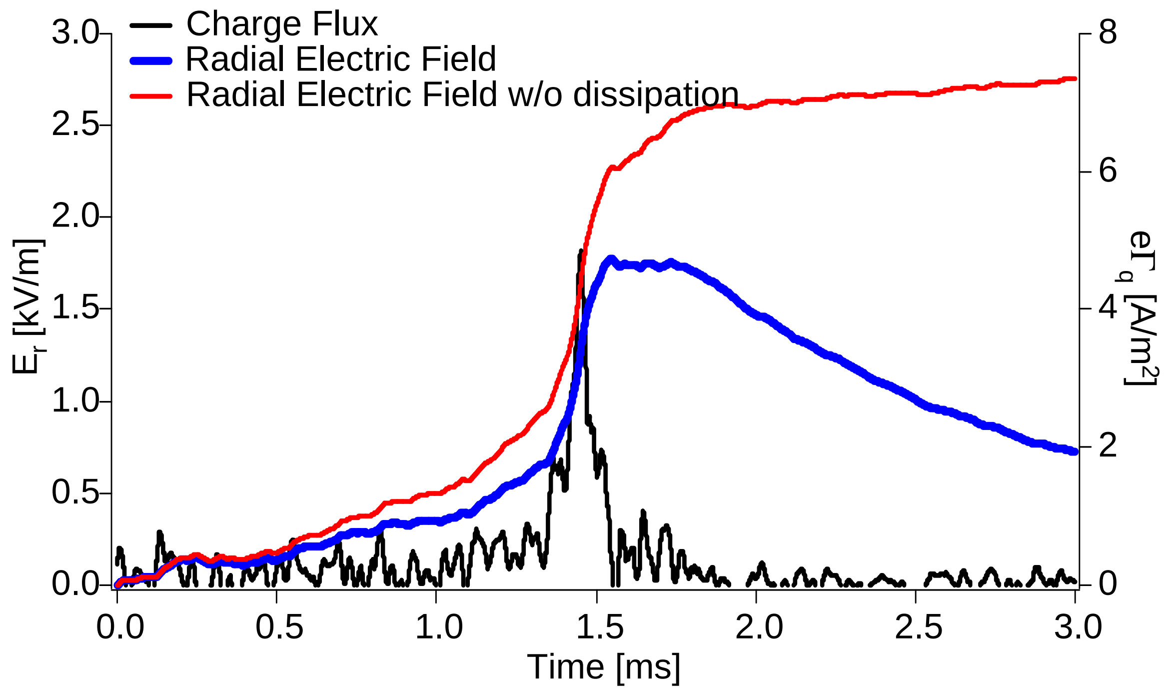
$$\downarrow$$

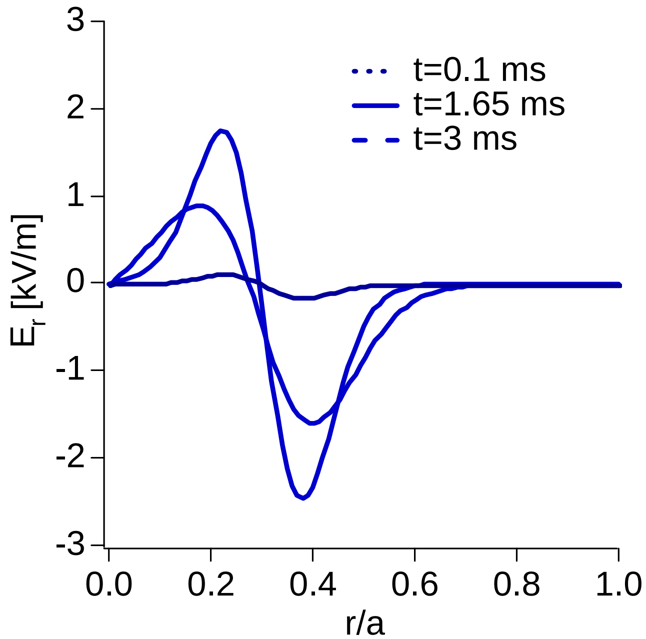
$$\Gamma_{x}^{i}-\Gamma_{x}^{e}=<\frac{\tilde{j}_{z}\tilde{B}_{x}}{eB_{0}}>+\frac{c}{eB_{0}}\frac{\partial}{\partial x}P_{xys}=\frac{1}{4\pi e}\frac{\partial E_{0x}}{\partial t}+\frac{n_{i}m_{i}c^{2}}{eB_{0}^{2}}\frac{\partial E_{0x}}{\partial t}$$

See: R. E.Waltz, Phys. Fluids, **25**,1269(1982);

$$\varepsilon_{0}\left(1+\left(\frac{c}{V_{A}}\right)^{2}\right)\frac{\partial E_{r}}{\partial t}\approx\varepsilon_{0}\left(\frac{c}{V_{A}}\right)^{2}\frac{\partial E_{r}}{\partial t}=\frac{<\tilde{j}_{//}\tilde{b}_{r}>}{B_{0}}+\frac{\mu_{\perp}^{*}}{B_{0}}\nabla^{2}V_{E\times B}$$

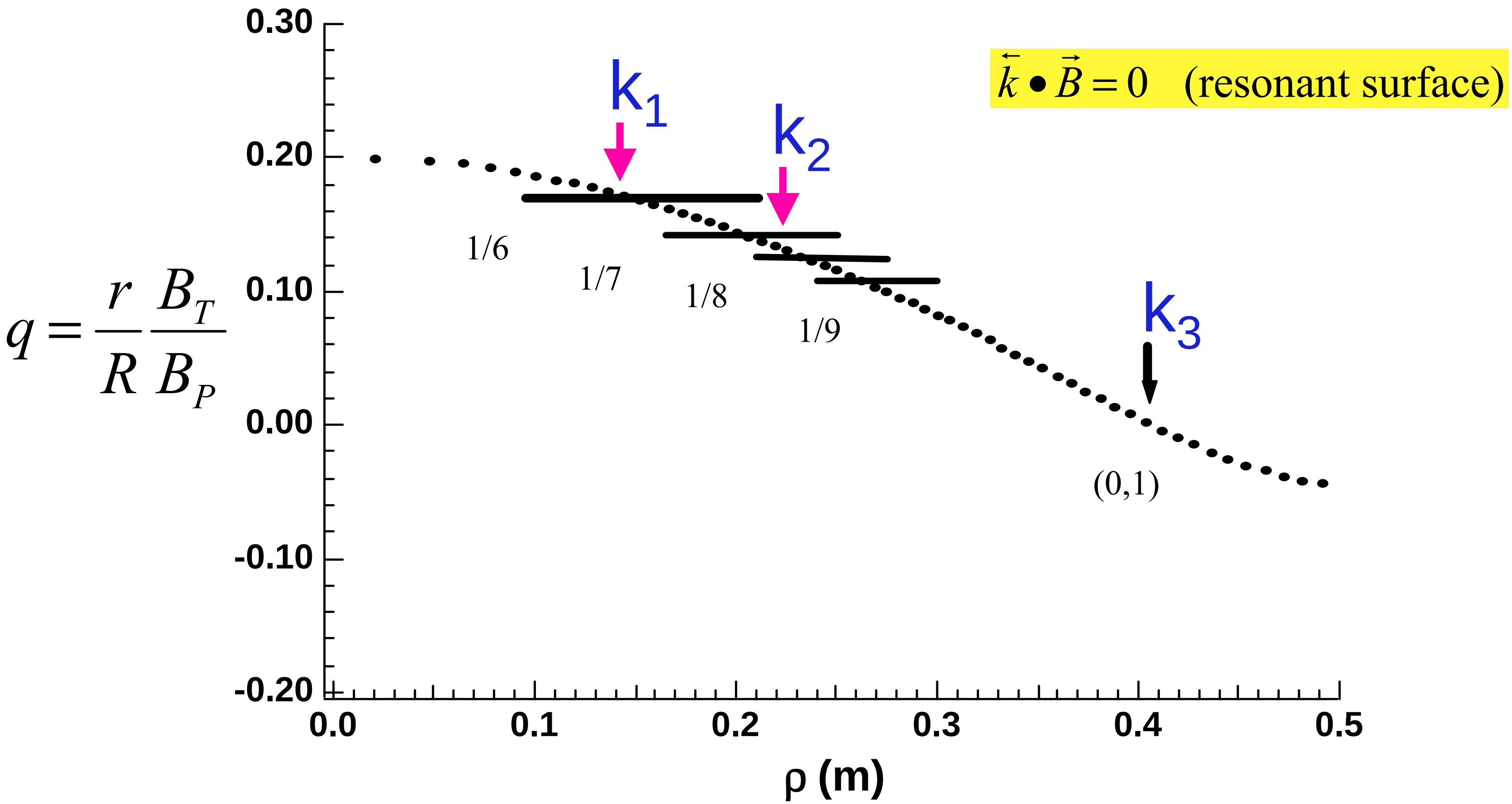
Electric Field Dynamics (with Collisional Dissipation)





(4)Effect of mode-mode interaction on <δJxδB> force

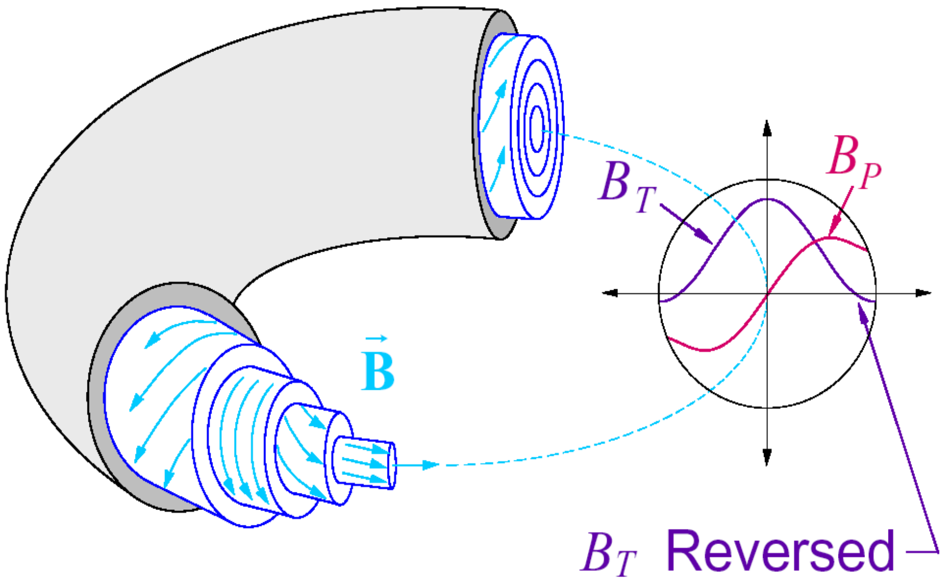
<δJxδB> force and charge transport result from nonlinear mode-mode interaction

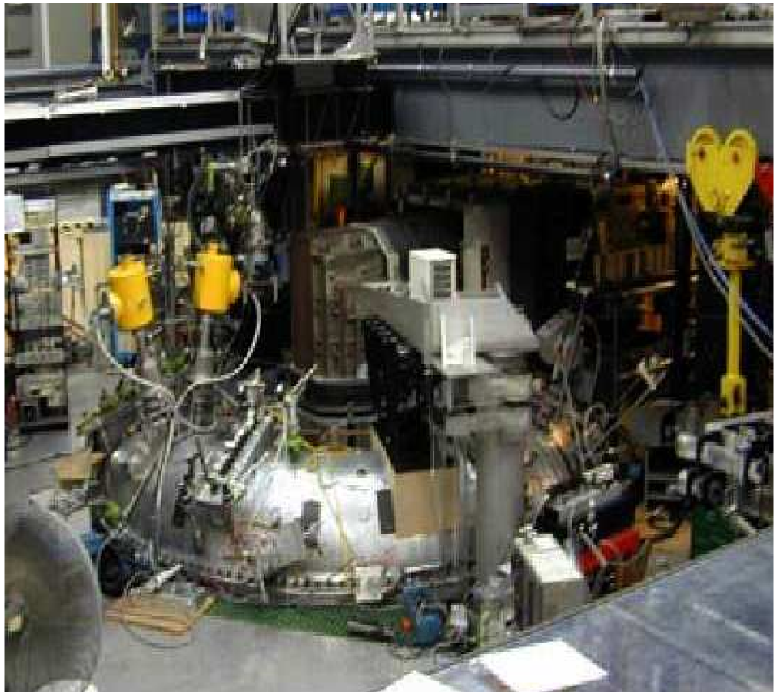


$$\vec{k}_1 \pm \vec{k}_2 = \vec{k}_3$$

Madison Symmetric Torus (MST) (Madison, Wisconsin)

MST Reversed-Field Pinch (RFP) is toroidal configuration with relatively weak toroidal magnetic field B_T (i.e., $B_T \sim B_p$)





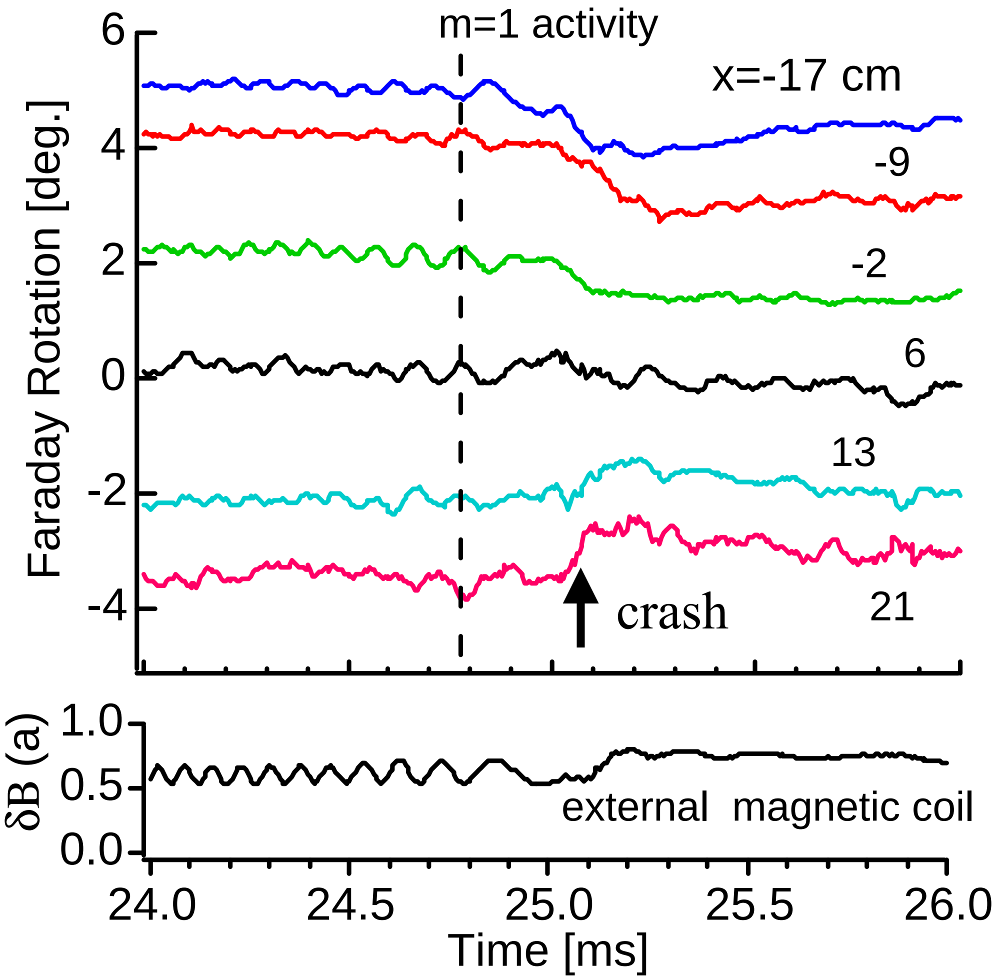
$$q(r) = \frac{r}{R}\frac{B_T}{B_P} < 1$$

R_0 = 1.5 m, a = 0.51 m, I_p < 600 kA

B_T ~ 3-4 kG, $n_e \sim 10^{19}$ $m^{-3,}$ T_{e0} < 1.3 keV

τ_E ~ 10 ms, $\beta = \langle p \rangle / B^2(a) = 15\%$

Measured Core Magnetic Fluctuations by Faraday rotation



Faraday Rotation $\Psi = c_F \int n\vec{B} \bullet d\vec{l}$

$$\Psi = \Psi_0 + \delta\Psi,\ \vec{B} = \vec{B}_0 + \delta\vec{B},\ n = n_0 + \delta n$$

$$\delta\Psi = c_F \int n_0 \delta\vec{B} \bullet d\vec{l} + c_F \int \delta n \vec{B}_0 \bullet d\vec{l}$$

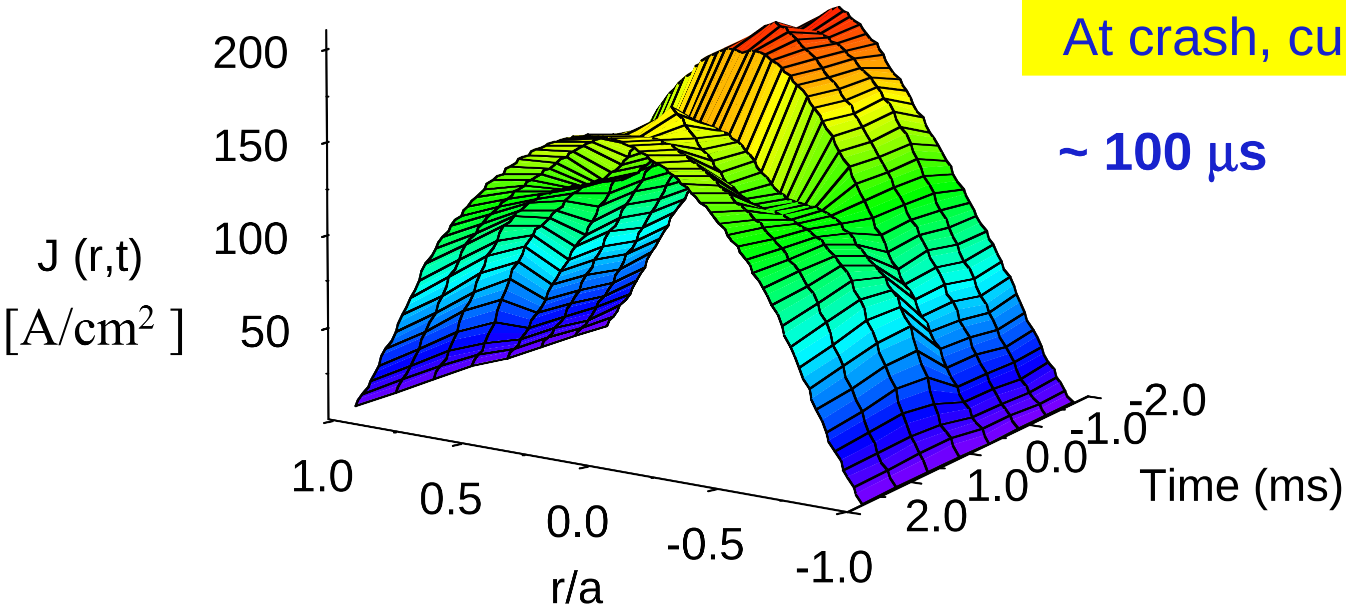
$$c_F \int \tilde{n} B_{z0}\, dz = c_F \int \tilde{n} B_\theta \cos\theta dz$$

$$\leq c_F \int (\mu_0 J(0) \frac{r}{2}) \frac{x}{r} \tilde{n} dz = c_F \frac{\mu_0 J(0)}{2} x \int \tilde{n} dz \rightarrow 0$$

$$\delta\Psi \approx c_F \int n_0 \delta\vec{B} \bullet d\vec{l}$$

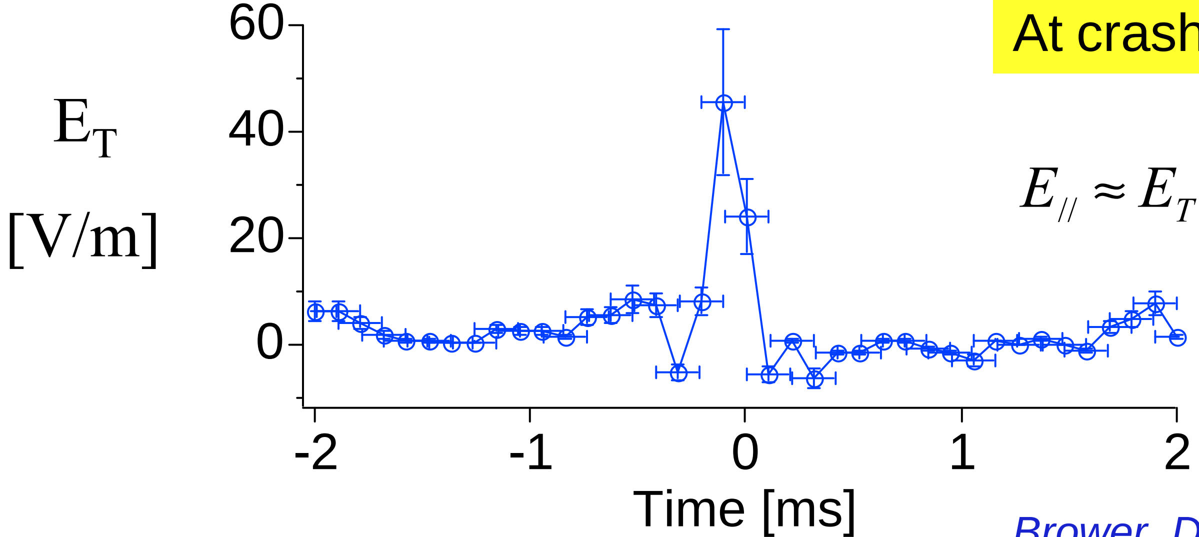
$$\delta\overline{B} = 33 \text{ [Gauss]}$$

(1) Measured Current Profile Relaxation



At crash, current profile flattens

~ 100 μs



At crash, electric field increases

$$E_{//} \approx E_T(r) = \frac{V_L}{2\pi R} - \int_r^a \frac{\partial}{\partial t} B_P(r',t)\,dr$$

$$E_{//} >> \eta J_{//}$$

*Brower, Ding, et al PRL,**88**,185005(2002)*

Hall Dynamo is balanced by Inducted Electric Field

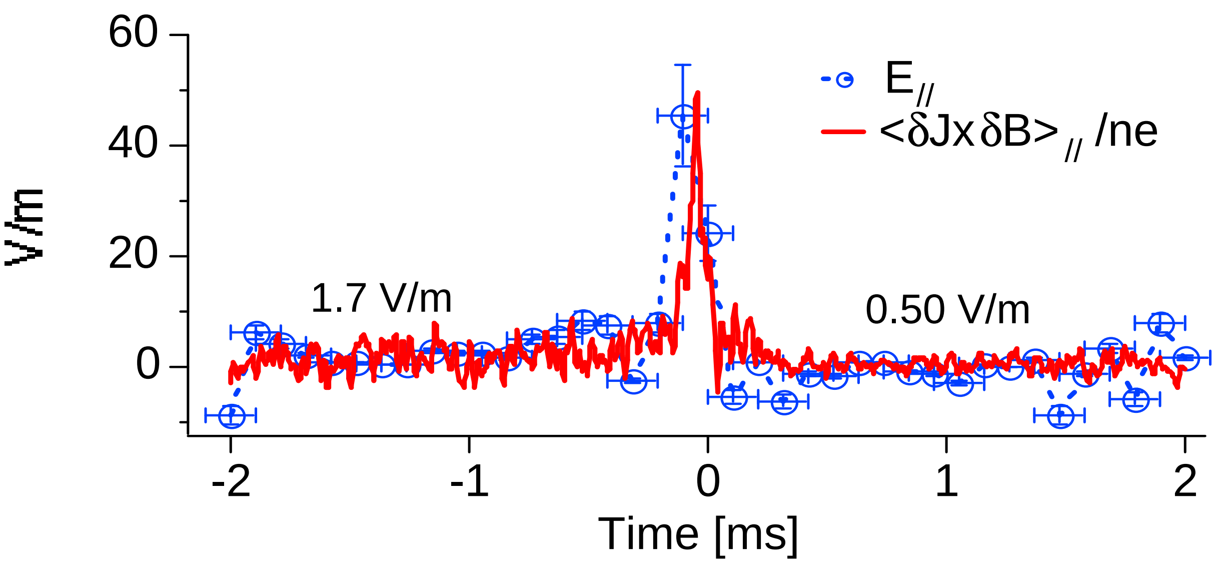
$$\eta_{//} < J >_{//} = < E >_{//} + \underbrace{< \delta\vec{v} \times \delta\vec{B} >_{//}}_{\text{MHD dynamo}} - \underbrace{< \delta\vec{J} \times \delta\vec{B} >_{//} / n_e e}_{\text{Hall Dynamo}}$$

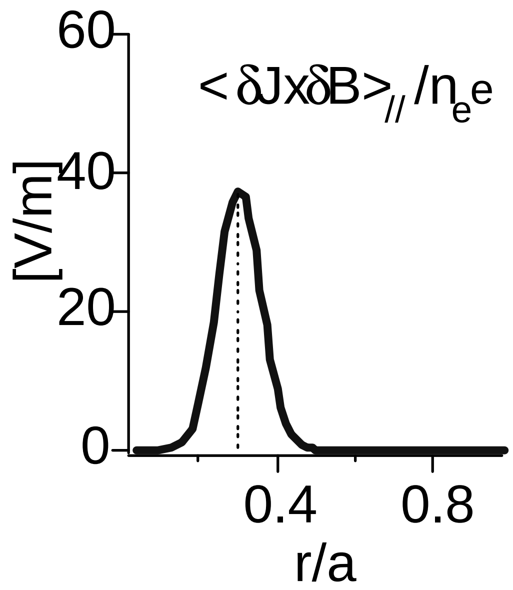
Parallel Mean Field Ohm's Law from 2-Fluid Theory

MHD dynamo

Hall Dynamo

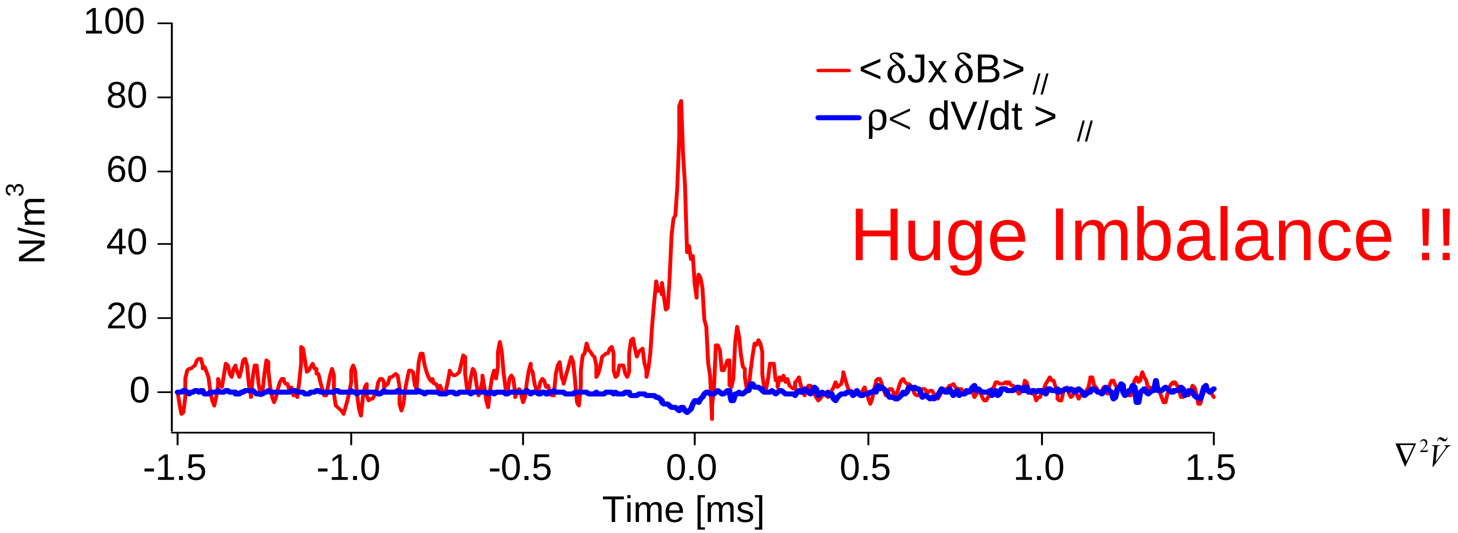
$$\frac{< \delta J \times \delta B >_{//}}{n_e e} \approx \frac{B_p}{B_T}\left(1 + \left(\frac{B_T}{B_p}\right)^2\right) < \delta j_\phi b_r >$$





*Ding, Brower, et al PRL,**93**,045002(2004)*

Ion Momentum (Torque) over Sawtooth Crash



$\nabla^2\tilde{V}$

$$< \rho \frac{\partial V}{\partial t} >_{//} + \rho < \tilde{V} \bullet \nabla \tilde{V} >_{//} \quad = \quad < \delta J \times \delta B >_{//} + \mu_{\perp}^{*} < \nabla^2 V >_{//}$$

$\sim 5\ \mathrm{N/m^3}$

$\delta V \leq 1\ \mathrm{km/s}$

$\Delta \mathrm{r} \geq 1\ \mathrm{cm}$

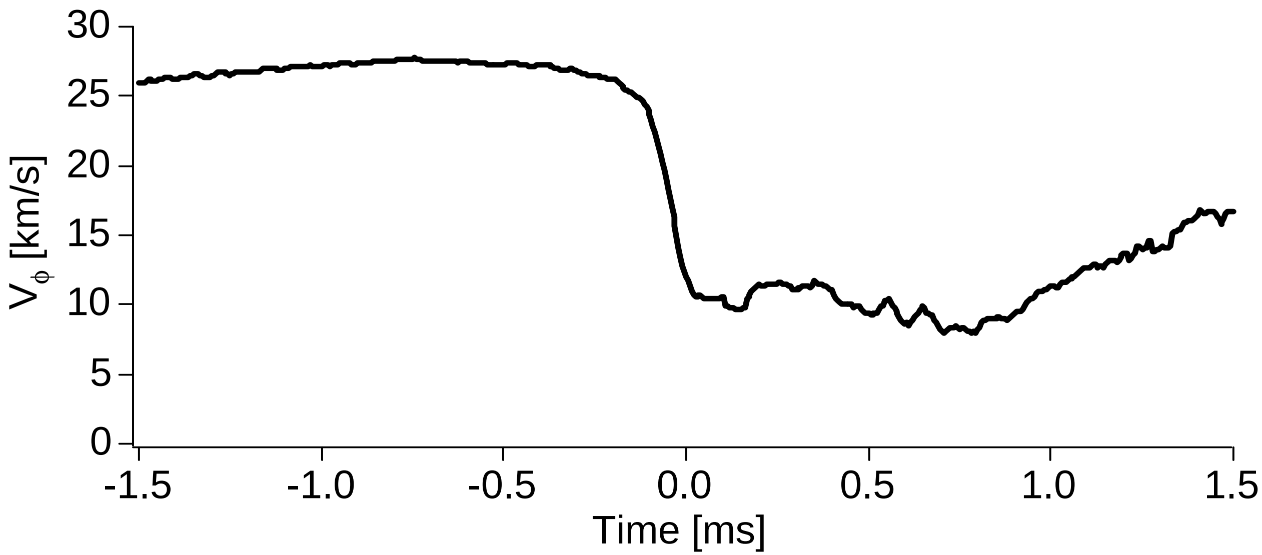
$\rho < \tilde{V} \bullet \nabla \tilde{V} > \leq 3\ \mathrm{N/m^3}$

$\sim 60\ \mathrm{N/m^3}$

$\mu_{\perp} \frac{V}{a^2} \sim -10^{-2} \sim 10^{-3}\ \mathrm{N/m^3}$

Classical dissipation

Ion Momentum Transport over Sawtooth Crash



At sawtooth crash ion momentum in the core drops much faster than classical viscous time.

Fluctuation-Induced Radial Charge Flux

$$\Gamma_q = \frac{<\tilde{j}_{//}\tilde{b}_r>}{eB} = \frac{<\tilde{j}_{//,i}\tilde{b}_r>}{eB} - \frac{<\tilde{j}_{//,e}\tilde{b}_r>}{eB}$$ charge flux

$$\vec{k}\bullet\vec{B} = \frac{m}{r_s}B_\theta + \frac{n}{R}B_\phi$$

On MST for a specified (m,n) mode

$$<\tilde{j}_{//}\tilde{b}_r> = <(\tilde{j}_\theta\frac{B_P}{B} + \tilde{j}_\phi\frac{B_T}{B})\tilde{b}_r>$$

$$\nabla\times\delta\vec{B} = \mu_0\delta\vec{J}$$

$$= \frac{R}{nB}(\vec{k}\bullet\vec{B}) < \frac{1}{r}\tilde{b}_r\frac{\partial}{\partial r}r\tilde{b}_\theta> = \frac{R}{nB}(\vec{k}\bullet\vec{B}) < \tilde{j}_\phi\tilde{b}_r>$$

$$\frac{|r-r_s|}{r_s} << 1$$

At mode resonant surface charge flux is zero,

but can be non-zero locally (near the resonant surface) to form charge filamentation*

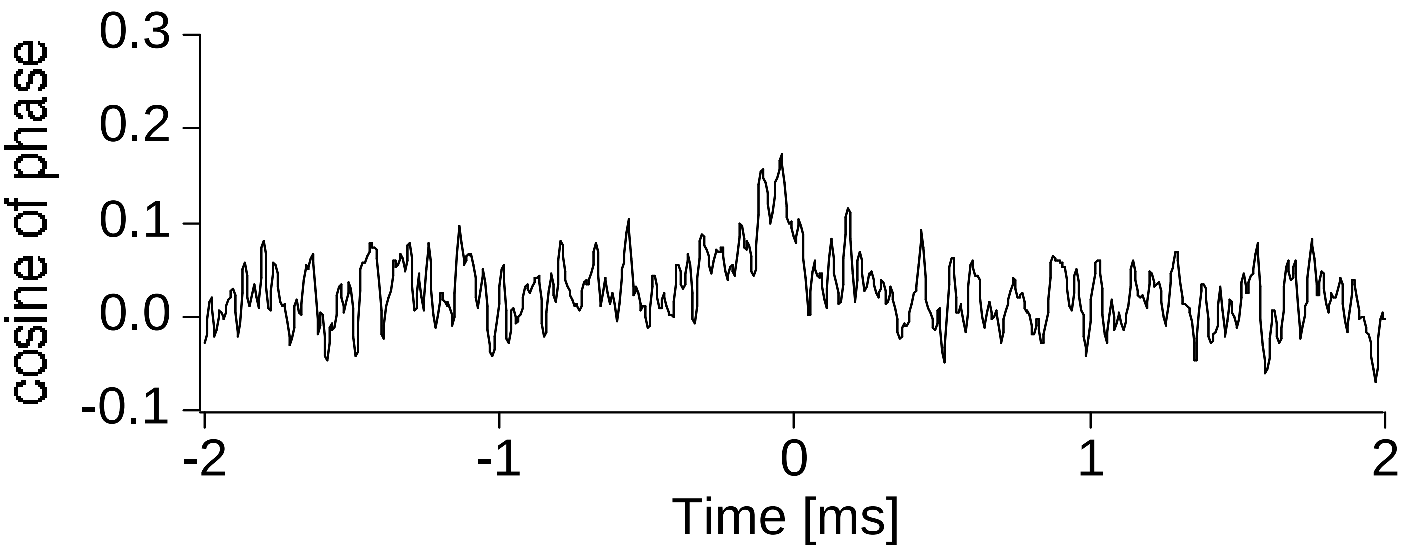
* R.E. Waltz, Phys.Fluid,**25**,1269(1982)

Phase between current and magnetic fluctuation

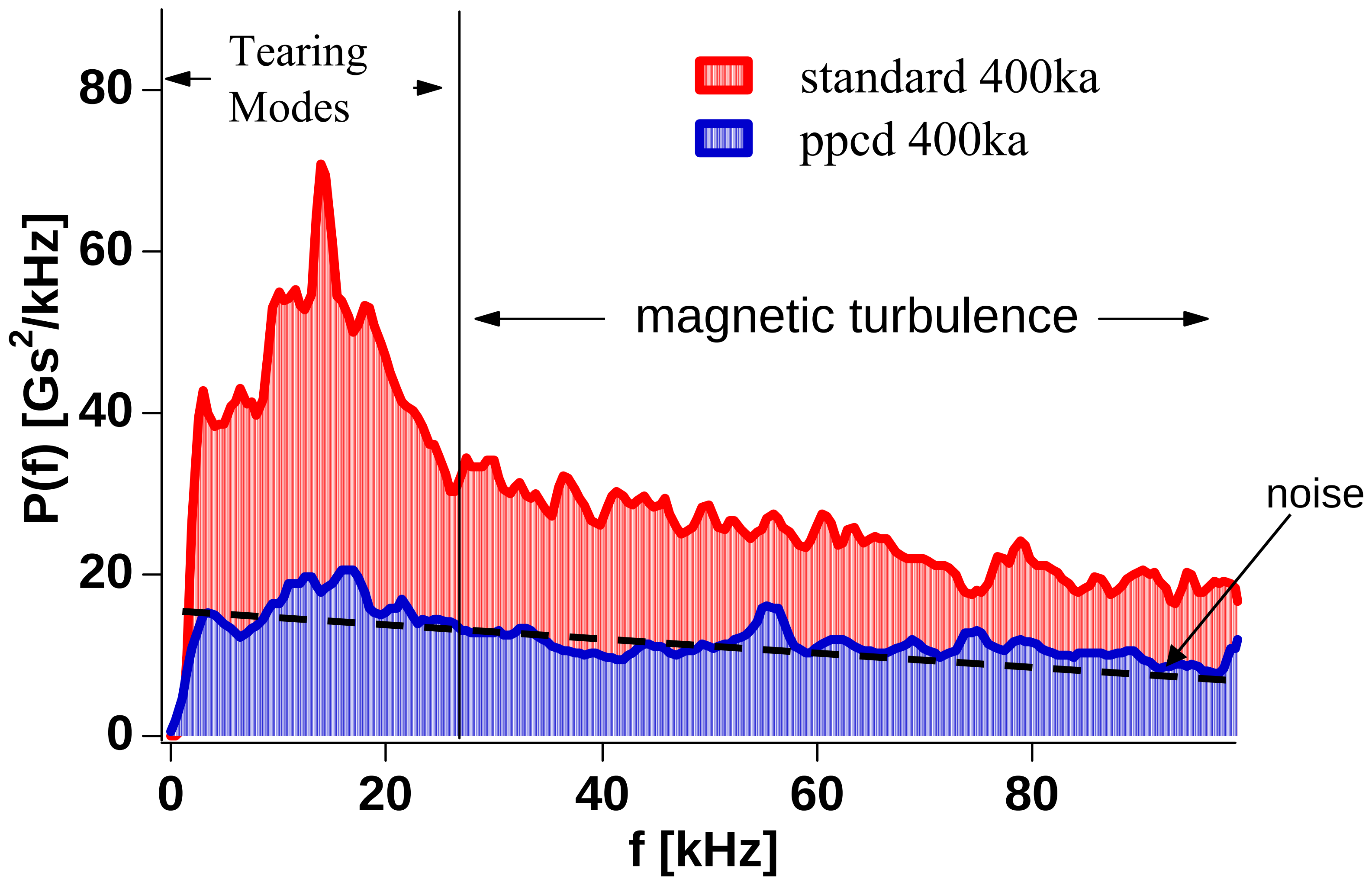
$$< \delta j_\phi(r_s)\delta b_r(r_s) >$$
$$= |\delta j_\phi(r_s)||\delta b_r(r_s)|\cos\Delta$$

$$\Delta = ph\_\delta j_\phi(r_s) - ph\_\delta b_r(r_s)$$
$$= ph\_\delta j_\phi(r_s) - ph\_\delta b_r(a)$$

Perpendicular global magnetic fluctuation has a constant phase



Magnetic Fluctuation Spectrum

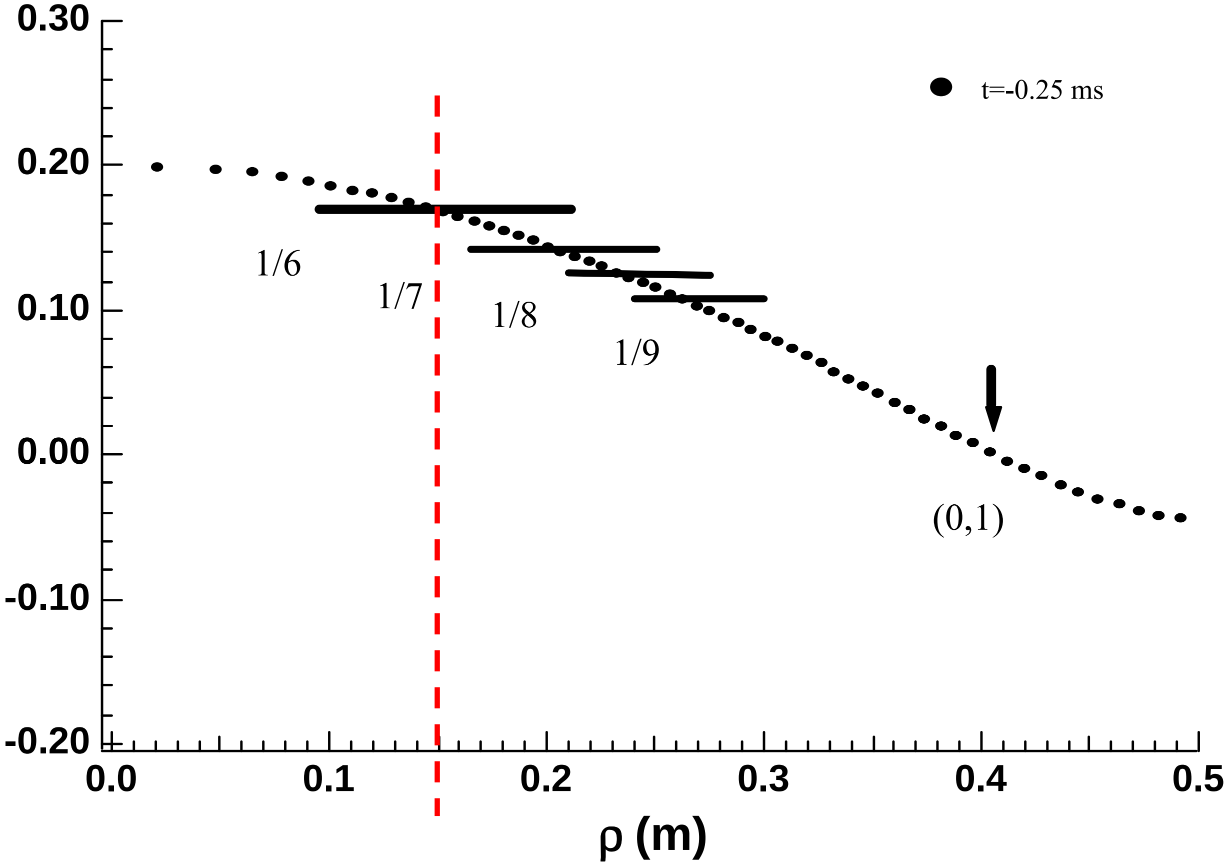


Tearing modes and broadband magnetic turbulence

RFP Safety Factor Profile

$T_e \sim T_i \sim 500$ eV

$$q = \frac{r}{R}\frac{B_T}{B_P}$$



Charge Transport and Radial Electric Field

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \vec{J} = 0, \quad \nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad \varepsilon_0 \frac{\partial E_r}{\partial t} = e(\Gamma_r^i - \Gamma_r^e)$$

$$\frac{< \tilde{j}_{//} \tilde{b}_r >}{B}$$

→ 10~(-120) [A/m^2] at the edge (probe, Neal Crocker, 2001)

→ 1~4 [A/m^2] at the core (FIR Faraday)

If Charge flux induced **only** by magnetic fluctuation

↓

$$\Delta \tilde{E}_r = \int \frac{< \tilde{j}_{//} \tilde{b}_r >}{\varepsilon_0 B} dt$$

→ 560 [MV/m]~6.7 [GV/m] at the edge (dt=0.5ms) >> T_e/a

→ 56 ~224 [MV/m] at the core (dt=0.5ms) >> T_e/a

Ion Polarization Drift

$$\frac{\partial\rho}{\partial t} + \nabla \bullet \vec{J} = 0, \qquad \nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0 \varepsilon_\perp}$$

See: R. E.Waltz, Phys. Fluids, **25**,1269(1982);

K.Itoh and S. Itoh, Plasma Phys. Control Fusion,**38**,1(1996).

$$\varepsilon_0 \varepsilon_\perp \frac{\partial E_r}{\partial t} = \frac{< \tilde{j}_{//} \tilde{b}_r >}{B}$$

$$\varepsilon_\perp = 1 + (\frac{c}{V_A})^2$$

Due to ion polarization current

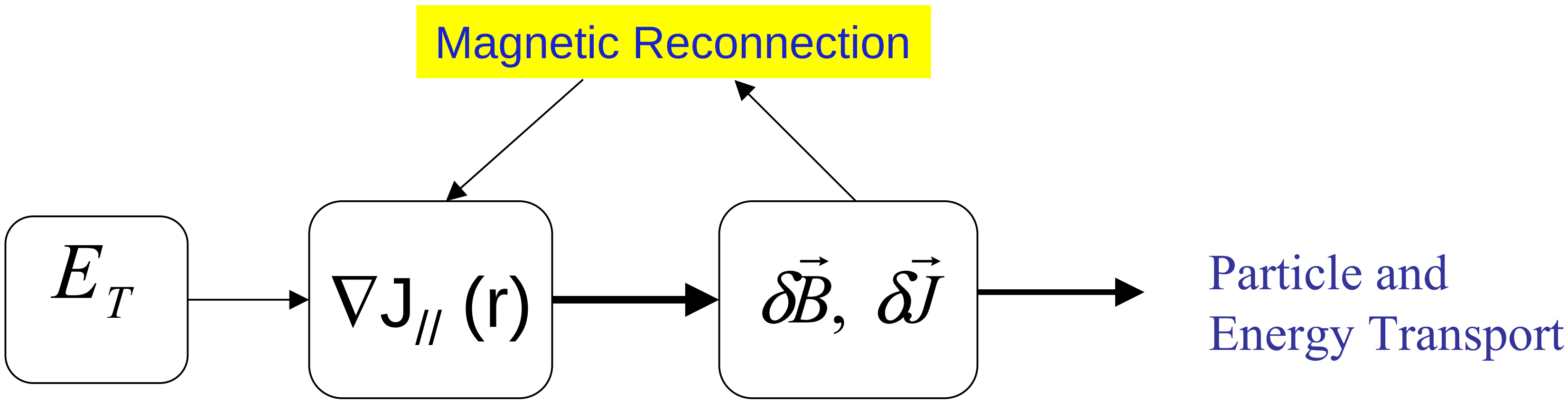
$$\Delta E_r = \left(\frac{1}{1 + \frac{c^2}{V_A^2}} \right) \int \frac{< \tilde{j}_{//} \tilde{b}_r >}{\varepsilon_0 B} dt \approx \left(\frac{V_A}{c} \right)^2 \int \frac{< \tilde{j}_{//} \tilde{b}_r >}{\varepsilon_0 B} dt$$

At reconnection, a radial electric field is established due to non-ambipolar transport,

but electric field is reduced by 10^4 due to shielding by the ion polarization drift.

Motivation

Magnetic fluctuations play an important role in magnetic reconnection in the laboratory plasma and astrophysical plasmas



$$\eta_{//}<J>_{//}=<E>_{//}+<\delta\vec{v}\times\delta\vec{B}>_{//}-<\delta\vec{J}\times\delta\vec{B}>_{//}/n_e e$$

MHD dynamo

Hall Dynamo

Outline

(1) Hall Dynamo: $\dfrac{<\delta J \times \delta B>}{ne}$

(2) Ion Momentum Balance: $<\delta J \times \delta B>$

(3) Magnetic Fluctuation-Induced Particle Transport; $\dfrac{<\delta J_{//} b_r>}{eB}$

- Maxwell Stress $<\frac{1}{r}\tilde{b}_r \frac{\partial}{\partial r} r\tilde{b}_\theta>$

All three processes are coupled through current density fluctuations

(4) Nonlinear mode-mode interaction

Identify the role of magnetic and current density fluctuations in particle and momentum transport